Institute of Education


Gender and participation in mathematics and further mathematics A-levels: a literature review for the Further Mathematics Support Programme

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IOE Supporting Advanced Mathematics Project
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## 1. Introduction

In preparing this report I have considered evidence from over 60 documents that relate to raising girls' participation in mathematics. These include published research papers and reports compiled by expert bodies that present an evidence base. Although research specifically addressing Further Mathematics A-level is rare, the last ten years have seen considerable efforts to synthesise and update knowledge from different research perspectives about the relationship between gender and participation. For this reason, the review process started with papers from 2008 onwards.

The large scale international tests such as the Trends in International Mathematics and Science Study (TIMSS 2003, 2007, 2011) and programmes of international student assessment (PISA 2003, 2006, 2009, 2012) have inspired studies comparing knowledge over time and across states and countries. This body of work throws light on arguments over environmental or biological causes of gender differences. In parallel, the statistical technique of meta-analysis has been used (largely in the United States) to pull together the results of similarly-constructed small -scale quantitative research enquiries. These help to establish overall patterns of significance and effect size, so that we can see what differences are stable over different contexts. In England, longitudinal or large-scale data has been used to track individual pupils' trajectories in mathematics up to A-level, in projects such as the DfE-funded Targeted Initiatives in Science and
 Mathematics Education (TISME) or Nuffield's ongoing project Rethinking the Value of A Level Mathematics Participation (that has not yet reported).

These studies give longitudinal information about how choices and attitudes change in individuals over time. This review also reports findings from research projects that are oneoff or smaller in scale but closely related to the UK mathematics education context.

To identify potential sources to include in the review, I followed three lines of enquiry based on knowledge of the field:

- Searching the British Education Index database for all relevant articles published since 2008 (using the search term "= post 2008 gender + mathematics + participation").
- Following citations in recent articles that characterise different approaches (e.g. starting with Hyde and Mertz (2009) for international studies and Alcock et al (2014) for personality factors).
- An internet search for relevant non-peer reviewed publications from organisations with an interest in mathematics education (Nuffield Foundation, Gender and Education, International Organization of Women and Mathematics Education, Ofsted, Institute of Physics, the research group Targeted Initiatives in Science and Mathematics Education (TISME)).

There were two main questions that drove the review, and these were used firstly to create a shortlist of relevant documents from their abstracts, and then to summarise and categorise their contribution. The shortlist was added to when further reading suggested that other sources were important to include. Summarising the documents also included a 'health check' judgement on their argument, evidence and relevance. This gave the following framework of questions:

1. What does this paper tell me about differences or similarities in female and male participation in advanced mathematics at age 16-18?

OR for less direct relevance:
What does this paper tell me about differences or similarities in female and male participation in mathematics at other ages?
2. What does this paper tell me about differences or similarities in factors that are thought to affect female and male participation in mathematics?
3. What recommendations are made about promotional events or teaching practices that increase participation in advanced mathematics, and what evidence is there for transferability to a Further Mathematics context?
4. Health check ( $0=$ not usable, $1=$ weak evidence or relevance, $2=$ strong and some relevance , 3 = directly relevant):
a. Are the arguments in the paper sound?
b. Is the paper informed by key thought in the field (bibliography and argument)?
c. Is there evidence that the findings can be generalised?
d. Is the context applicable to FMSP?

The following report addresses themes that arose from this analysis.

## 2. Why pay attention to gender in mathematics education?

This is not a question that can be determined by research evidence, yet almost every research paper addresses it. All the papers reviewed show a concern for social, economic and institutional injustices that result from women's unequal participation in advanced mathematics. Many papers also argue that their nation's economic advantage relies on increasing the proportion of the population with mathematical skills. From this perspective, girls who do not follow STEM courses are a potential source for recruiting more mathematicians, and hence their participation deserves scrutiny.

Differential take-up in mathematical and scientific careers is widespread, although the time that these differences appear in education varies. By the age of 15,51 out of 54 countries in PISA 2006 had a statistically significant difference in the proportion of boys and girls planning a career in engineering or computing, all towards boys; with the UK near the OECD averages ( $5 \%$ of girls and $18 \%$ of boys) (OECD, 2012). The latest school data for England shows that 20.4\% of the females in the 2012-13 A-level cohort entered for the mathematics A-level examination, compared to $37.4 \%$ of boys, nearly twice as many (although there are more girls in the cohort so the ratio within mathematics lesson is closer to 1:1.5). For Further Mathematics, there are nearly three times as many boys, with $2.4 \%$ of the girls entered for A-level, compared to $7.4 \%$ of boys (DfE, 2014). In contrast, in the United States, boys' and girls' participation in optional calculus courses at high school has been equal for over ten years (College Board, 2013) and nearly 48\% of mathematics-major college degrees are awarded to women (Ceci \& Williams, 2010b). These equal rates in the US do not (yet) persist into later study, dropping to $29 \%$ of PhDs. However they give us an indication that under-representation at 16-18 has been challenged in cultures that are close to our own. Thus comparative research, social justice and economic imperatives combine convincingly to suggest that girls' choices about mathematics and science should be a policy focus. There is also a significant gender bias - but in favour of girls - in participation in subjects such as language or careers such as veterinary medicine, but this is not seen to have the limiting implications for boys that biased mathematics participation has for girls.

There is a counter-argument or caveat discussed in the more thoughtful papers, which is that the amount of research attention paid to gender differences far outweighs the significance of the findings. There is a historical legacy of interest in gender, which guarantees an audience. Perhaps more importantly, it is an easy variable for researchers to work with. Collecting data on gender has no obvious problems of reliability or validity across time or across social or national contexts. It is not seen as intrusive and yet seems relevant to individuals' performance. For example, a recent research project aiming to understand participation in mathematics and physics found that some schools were unwilling to ask pupils survey questions that indicated social class but had no problems with gender (Mujtaba \& Reiss, 2013). Together, the audience interest and ease of collection encourage research in which data is routinely analysed by gender without an obvious hypothesis but in
the hope of reporting whenever the male and female populations are different. This approach keeps attention on gender when there are much larger differences in mathematics performance and trajectories, for example between students in rich and poor countries (Kane \& Mertz, 2012), rural and urban communities (Wei et al., 2012) and in the UK between students of different socioeconomic status (Noyes, 2009; Ofsted, 2014; The Royal Society, 2008; Strand, 2011). This propensity to look for the gender angle is worth bearing in mind when interpreting research, and may be an unhelpful focus of interventions (see section 9).

As mathematicians, we know that statistical significance establishes our confidence in any assertion that male and female populations have different means on a given measure. In the discussion below I have reported quantitative research findings as significant only if they are reported as statistically significant at a $1 \%$ level: there is less than a $1 \%$ probability that the perceived difference occurred because of the random nature of the sample taken from girls and boys populations with the same mean scores. In research involving thousands of students (such as PISA, TIMSS and UPMAP) even small differences are statistically significant: we can be very confident that there is a very small difference in the averages. Effect size is reported in research so that readers can start to judge the implications of that difference by comparing it to the variability within the data and then to other findings. The most common measure, Cohen's $d$, uses the difference of means divided by a standard deviation to produce a standardised difference. Effect sizes of 0.2 are considered small: present but hardy visible, comparable to the average height difference between 15 -and 16year old girls. Effect sizes of 0.5 are considered medium, comparable to the height difference between 14 -and 16 -year old girls, or one grade at GCSE; and effect sizes of 0.8 are considered large (Coe, 2002).

There are still arguments about implications. Some researchers argue that a tiny effect size can nonetheless make a difference to many people depending on context. For example raising US girls' scores on college entrance mathematics examinations to the boys' mean score could result in thousands more girls qualifying for a STEM subject (Ceci \& Williams, 2010b). Post-structural research argues that even finding no difference in male and female performance does not mean that mathematics is not gendered. They point to the many ways in which mathematics is connected through language and structures to ideas that are themselves aligned with masculinity (Mendick, 2006) and to the salience of gender in young adults' decision making. This means that the boys and girls doing mathematics and further mathematics A-levels have different ways of making sense of that 'same' experience to themselves and in relation to other people (Smith, 2010).

Wiliam (2010) reminds us to judge good research by the validity of what is being examined and by the researchers' attention to competing explanations of the same results. In a recent study, Alcock et al. (2014) have illustrated this approach. They considered whether the gender of 89 undergraduate mathematics students was related to their grades and selfreported learning approaches, and in the same survey they assessed for 'personality factors' using a psychological model that scores people on conscientiousness, extroversion,
agreeableness, neuroticism, and openness to experience. As expected from previous research, these personality factors showed an association with the students' gender, with women scoring slightly higher on Agreeableness, Conscientiousness, and Neuroticism (with effect sizes of $d=0.694,0.551$, and 0.570 ). The techniques of multilevel modelling allowed the authors to assess the contribution of gender after controlling for the effect of personality factors and, conversely, for each personality factor after controlling for the effect of gender. They found that personality type accounted for significantly more variance in undergraduates' achievement and behaviours than did gender. In particular achievement was correlated in both males and females with conscientiousness, which measures the tendency to show self-discipline and regulate impulsive behaviours. It certainly makes sense that self-disciplined undergraduates achieve highly. The authors' wider contribution has been to illustrate that gender can seem a valid explanatory factor when it is actually a proxy for other related factors such as personality which are easier (though not easy) to change. Although a proxy is superficially useful, it obscures the variability within gender groups, for example ignoring patterns in how disagreeable girls or conscientious boys do mathematics. The message from this paper is that initial findings of gender differences should motivate more research to find out what lies behind them and whether there are explanatory factors that are susceptible to change through learning.

The next two sections address one of the key overall questions of the review: what are the recent international findings on differences and similarities in male and female participation in mathematics? Section 3 introduces the range of factors that have been shown to affect participation in A-level mathematics, amongst which the most important is prior attainment at GCSE, followed by gender. Section 4 considers the evidence related to boys' and girls' achievements in mathematics. Following this there are five sections related to gender differences in factors associated with participation and recommendations of how schools and teachers might respond to these. These address the second key question: what recommendations are made about promotional initiatives or teaching practices that increase participation in advanced mathematics, and what evidence is there for transferability to a Further Mathematics context?

## 3. Factors that affect participation in A-level mathematics

There are five factors that are widely found to affect students' intentions to study mathematics at A-level that could be influenced by school practices. These are prior attainment in mathematics, enjoyment, perceived competence, interest in mathematics and awareness of the utility of mathematics for supporting access to other areas. Student background factors of gender, ethnicity and socioeconomic status interact with these and are also significant in affecting participation (Boaler, Altendorff, \& Kent, 2011; Strand, 2011; Tripney et al., 2010). The focus in what follows is claims that are made about gender.

The national pupil database means that it is possible to track background information for large numbers of students who have entered A-level mathematics or further mathematics
examinations. Noyes (2009) used this database for a cohort of 41,000 A-level students in the Midlands regions and found that prior attainment at GCSE mathematics was the single most significant predictor of continuing to $A$-level. $82 \%$ of students with an $A^{*}$ in mathematics continued to AS-level mathematics or beyond, but only $53 \%$ with an A and $16.8 \%$ of those with a grade B. The difference in participation for A and A* grades is thought to result from student choice rather than school guidance. The interaction with gender was marked and again results from student choice. Girls and boys achieve very similarly at GCSE, with differences of less than $1 \%$ in the proportions of boys and girls getting each grade in 2013 (DfE, 2014b). However, given the same grade, boys in Noyes's sample were more likely than girls to continue mathematics to A-level. The disparity got wider for lower grades ( $86.5 \%$ of boys compared with $77.4 \%$ of girls with $\mathrm{A}^{*}$ moving to $23.1 \%$ vs $11.5 \%$ with grade B). Noyes's finding has been supported by later data analyses (Department for Education, 2011; Hodgen, 2013; Mujtaba \& Reiss, in preparation). This suggests that there may be large numbers of girls with grades A or B in mathematics GCSE who might be encouraged to choose mathematics A-level.

Relative attainment is recognised as another factor in this choice. Noyes found that students are more likely to take part in mathematics A-level if their mathematics grade was higher than their other GCSE grades. This is consistent with the perspective found amongst A-level students and teachers that you have to be unusually 'clever' to continue with mathematics (Matthews \& Pepper, 2007). Although the image of a specialist is familiar in mathematics, this preference also applies to other subjects. Relative attainment at GCSE is significant for participation in physical science A-levels (The Royal Society, 2008) and for choosing advanced mathematics courses in the United States (Diane Halpern et al., 2007). The evaluation of A-level changes in 2010 reported that students are increasingly choosing to continue with the AS-level subjects which they find 'easiest', based on prior attainment and experience (AlphaPlus Consultancy Ltd, 2012).

This is relevant to gender differences because more girls than boys gain the top GCSE grades in England (with twice as many getting A or A* in English Language for example) so that academic girls' choice patterns reflect the wider possibilities that are open to them as well as their positioning as all-rounders rather than specialists (Sullivan, 2009). We can ask whether feeling qualified in a broader range of subjects affects girls' decisions about mathematics beyond mere availability. Thoman et al. (2014) surveyed women US college students fortnightly over a whole semester and found that most students felt a sense of belonging in their mathematics courses that was independent of their sense of belonging in humanities. However students who started to feel that they were lower achievers in mathematics than they were in humanities, and who valued their peers' opinions, were affected by this contrast and lost interest in mathematics. The message from these findings are that we need to be careful about presenting participation in mathematics as only for very high-attaining students because girls' choices already conform to this pattern, more so than boys' (see §9 below for a discussion of self-concept). Both boys and girls who have other viable options need support to get over initial problems and continue in mathematics.

After prior attainment and gender, the factors usually found to be significant for girls choosing mathematics A-level are interest and/or enjoyment. Brown, Brown and Bibby's (2008) study of nearly 2000 year 11s reported that girls are more likely than boys to give interest and/or enjoyment as a reason for their STEM-related subject choices, with $50 \%$ of girls compared to $30 \%$ of boys. Boys are more likely to cite instead that mathematics is easier than other subjects. This difference was rated as one of the most robust research findings in Tripney at al.'s (2010) systematic literature review, underpinned by repeated primary empirical research from OECD countries. The importance of enjoying your study is also underlined by qualitative work that examines girls' accounts of classroom experiences (Solomon, 2007) and A-level choices (Hernandez-Martinez et al., 2008; Mendick, 2006; Smith, 2010).

The UPMAP project (Understanding Participation in Mathematics and Physics) surveyed nearly 11,000 year 8 (age 13) and year 10 (age 15) students from 133 English schools during the academic year 2008-2009 and considered enjoyment through a range of questions related to mathematics lessons and mathematics teachers (Mujtaba \& Reiss, 2013). They used multilevel modelling to find the contribution of any one variable while controlling for others. Students' intentions to continue with mathematics were significantly associated with high scores on perceptions of mathematics lessons, emotional response to mathematics lessons and perceptions of mathematics teachers (alongside extrinsic material gain and encouragement by family which I discuss in sections 5 and 9).

Boys scored higher than girls in their perceptions and emotional response to mathematics lessons, with small effect sizes of 0.15 and 0.09 respectively, and there was no difference overall in their perceptions of teachers. Year 10 students had more negative perceptions than younger students. Although the effect size by gender alone is very small, a feature of this research is its comparison of effect sizes across all four subgroups of boys/girls (B/G) with high/low (H/L) mathematics aspirations. Separating by subgroups showed medium effects of subgroup membership on the two enjoyment indicators ( 0.42 for perceptions and 0.28 for emotional response), showing that enjoyment is even more important for mathematics aspirations for girls than it is for boys. The highest means for both are for high mathematics aspiration boys (HB) and the lowest for low aspiration girls (LG): HB>HG>LB>LG. This research is supported by a smaller-scale study in Welsh schools (Cann, 2009), and by the PISA 2012 overall findings that fewer 15-year old girls than boys report enjoying mathematics (OECD, 2014). Together these research papers add up to show convincingly that from age 13 to 16 both girls and boys are more likely to think about continuing with mathematics if they enjoy it, and that this factor is more important for girls, while they report enjoying mathematics slightly less than boys do.

Having good examination results and enjoying mathematics make a difference to students choosing mathematics. If we want to encourage boys and girls to choose mathematics Alevel we need to improve these factors. Although the positive effect is obvious, it is complicated by teachers' and students' knowledge that the transition from GCSE to A-level usually involves an academic struggle and a dip in performance. The research suggests
that if we don't pay attention to supporting students when they are not achieving highly or enjoying mathematics then we will lose more girls than boys. However enjoyment is not an isolated factor. In particular the experience of Science colleagues has been that recent GCSE reforms have increased girls' enjoyment of science at GCSE but they still report feeling that science A-levels and careers are 'not for me' (Archer, DeWitt, \& Wong, 2014). As in the UPMAP study, this points to the importance of considering how enjoyment interacts with other factors, particularly those concerning motivations, encouragement and students' self-concept in mathematics (their reported feelings of how well they are doing).

It would be interesting to know whether equal proportions of girls and boys drop out of Alevel mathematics in the first few weeks of the course, or stop after AS-level. I have not found any published research that traces these decisions in school by gender. The data linking AS to A2 results is complicated as students do not necessarily take an AS-level in year 12 or certificate their results. Noyes's (2009) data showed $9 \%$ of girls in his sample ended up with only an AS-level mathematics, compared with $12 \%$ of boys, but $18 \%$ of girls ended up with a full A-level compared to $28 \%$ of boys, compatible with more girls than boys leaving after AS-level. However, DfE data from 2013 shows no clear difference in the proportions of girls and boys taking AS-level and A-level mathematics (DfE, 2014a).

The messages from these findings are:

- we need to be careful about presenting mathematics as only for those getting the highest grades, because this reinforces a pattern in girls' participation where girls with GCSE grades As and Bs are even less well represented at A-level than girls with A*s.
- the relationship between enjoying mathematics and intentions to continue mathematics post-16 is more marked for girls.
- both boys and girls need support to get over initial problems and continue in mathematics if they have other viable options.


## 4. Gender differences in mathematics performance

Gender performance in mathematics has been investigated on a large scale in two ways. The first is through mathematics assessments sat by thousands of students. PISA and TIMSS, national grade-by-grade tests and college entrance tests in the US and public examinations in the UK are examples of these. The second is by meta-analyses compiling the data of smaller research studies in individual laboratories and schools. In both cases the scale of the research is only valuable if we agree that the tests and studies are measuring essentially the same construct over all the sites and test occasions (Wiliam, 2010). Although they are open to critique, the large repeated international and national assessments provide evidence that researchers have used to test and refine hypotheses over time.

If there is a construct such as overall mathematics performance being measured by all these studies, then it is the same for girls and boys. Data has been analysed from TIMSS or PISA 2003 (Else-Quest, Hyde, \& Linn, 2010), TIMSS 2007 and PISA 2009 (Kane \& Mertz, 2012) and PISA 2012 (OECD, 2014). There is considerable variation between countries, with many more countries whose boys do slightly better than girls in mathematics rather than vice versa. No statistically significant gender gap existed overall in the mean scores of fourth and eighth graders on the 2003 and 2007 TIMSS (Kane \& Mertz, 2012). Where statistically significant differences have been found, they have very small effect sizes. For PISA 2012, the mean gender difference of 12 points (on the 1000 point scale) for the UK has an effect size of 0.13 , close to the OECD average of 11 points with effect size of 0.12 . PISA uses four content subscales (change and relationships, space and shape, quantity and uncertainty and data) and three process subscales (formulating situations mathematically process; employing mathematical concepts, facts, procedures, and reasoning process; an interpreting, applying and evaluating mathematical outcomes). The pattern is similar for all of these subscales: gender differences are not significant for Northern Ireland, and the effect sizes are less than 0.2 for England Wales.

In the US, Hyde et al. (2008) analysed the school assessments from 7 million students in 10 states in 10 grades between ages 7 and 17 and found trivial gender differences in mathematics performance (effect sizes $<0.06$ ). This confirmed their earlier results from a 1990 statistical meta-analysis, combining the results of 100 trials involving 3 million individuals from the US, Canada and Australia that found only a tiny effect size in favour of better female performance ( $\mathrm{d}=-0.05$ ). The picture of small differences is the same for both GCSE and A-level mathematics in England and Wales, although this is often reported as girls having higher pass rates (Department for Education, 2011). In 2012 and 2013, the percentages of boys and girls getting each GCSE grade $A^{*}$ to $E$ differed by less than 1\%. Differences in the percentages of boys and girls who took A-level are slightly bigger, with 3$4 \%$ more boys getting an $A^{*}$ but $2-3 \%$ more girls getting an $A, 2 \%$ more girls getting a $B$ and other differences less than $1 \%$. Although DfE data do not show effect sizes, these overall differences are small, and support the research findings that on average girls and boys achieve equally well in mathematics.

There are two aspects of mathematics performance that have remained of interest. One was a finding from a 1990 meta-analysis that boys performed better than girls on questions involving complex problem solving. Interpretation of this result was difficult at the time as US girls took fewer advanced mathematics courses aged 16-18. The same researchers returned to this result after US participation rates in advanced mathematics courses became equal, and found that US national test data of 17 year olds showed no significant differences in tests that include complex problems (Hyde \& Mertz, 2009) suggesting that the original difference was a result of differences in experience. PISA 2012 has focussed on problem solving in 15-year olds (although not complex problem solving in Hyde's terms) and shows UK girls and boys performing equally well, both above the OECD average. This illustrates the contribution that research can make to refining and testing hypotheses about gender differences, and it no longer seems likely that this difference exists.

The second aspect is known as the greater male variability hypothesis. The spread of boys' results in mathematics is greater than for girls, and hence there are more boys than girls in the top and bottom $5 \%$ and $1 \%$ of any assessment. This is found in the large international tests and US college entrance tests as well as in assessments that identify gifted mathematicians (Halpern et al., 2007; Heilbronner, 2013; OECD, 2014). However this result is not stable across time, countries or ethnic groups. In US tests the greater variance of boys compared to girls has reduced over time, getting closer to a ratio of 1, but remaining a significant difference (J. Hyde \& Mertz, 2009). On 2007 TIMSS items the UK is average among OECD countries with a ratio of male to female variability between 1.05 and 1.12 (Kane \& Mertz, 2012). Hence this is a hypothesis that research is still looking to test, and much of the interest is in the extremes of ability such as mathematics olympiad teams and precociously gifted youth. In the UK the greater male variability hypothesis is compatible with the slight over-representation of girls within the middle A-D grades at GCSE ( $<1 \%$ difference per grade) and the slight over-representation of boys with an $\mathrm{A}^{*}$ grade (7.1\% compared to $6.7 \%$ in 2013) but it does not rule out other contextual explanations. However, because the variance ratio is close to 1 , even if the hypothesis is found to hold, it seems very unlikely to account for male over-representation in A-level mathematics and further mathematics. In the US context, theoretical models have shown that the known effect is not large enough to account for the actual differences in STEM participation at college level (Ceci \& Williams, 2010a). The message from research is that there are slightly more boys than girls who perform either very well or very badly in mathematics tests, but we do not know why nor whether this is a result that will continue to change.

International test data has offered researchers the opportunity to test hypotheses that relate gender differences in mathematics to biological factors (that would be constant between countries and over time) or environmental/ cultural factors (that could vary in predictable ways). The between-country variation in gender differences both at the mean and at the extremes of performance throws doubt on purely biological explanations. Work on cultural hypotheses continues. One interesting hypothesis that has since been rejected was the finding that the gender gap in mathematics in PISA 2003 data was significantly associated with the GGI index used by the World Economic Forum to indicate country's gender inequality. An initial study found that the more unequal a country's society, the greater the gap in gender performance. However, this gap due to gender inequity hypothesis was rejected when the finding was not reproduced in the TIMSS 2003, 2007 or the PISA 2009 tests. Instead, researchers found that both girls and boys were found to perform better in more gender-equal countries (Kane \& Mertz, 2012).

There is one aspect of mathematics where boys are consistently found to excel, and that is in tasks involving interpretation of 2-D drawings of 3-D objects and mental rotation of these images. The biological and psychological evidence for this was extensively reviewed by Halpern et al. to underpin a US report promoting girls' participation in mathematics and science. The clear definition of the task type has helped establish this result as robust, stable over time and countries. There is similar agreement that girls outperform boys in writing tasks throughout school, an effect which is larger and similarly stable. Girls are also
found on average to have a stronger episodic memory than boys: they remember what they experienced. The review finds that all three differences are compatible with contemporary neuroscience findings about brain structure and function, but warns against attributing them solely to either biological or environmental factors (Diane Halpern et al., 2007, p29). One reason for their caution is that experiments (with specialists such as taxi drivers) show that practice physically changes the brain's structure. Hence modern neuroscience tells us that nature and nurture are not as distinct as once thought, and we do not yet know enough about how brains change through education and childhood to guide policy (Fine, JordanYoung, Kaiser, \& Rippon, 2013; The Royal Society, 2014).

Mental rotation is an important skill for engineering, architecture, geometry, craft or construction work, and features in cognitive aptitude tests for non-verbal reasoning. Halpern et al. (2007) point to evidence from engineering courses that it is a skill that can be taught when needed and that it develops through practice, for example with video games. Research is ongoing to identify other specific aspects of mathematics on which girls and boys will consistently perform differently, but there are none with the same weight of evidence as mental rotation.

The messages from this research are:

- Recent international evidence suggests that on average girls and boys now perform equally well in mathematics.
- There is still a small difference in the spread of girls' and boys' attainment, with more boys at both extremes of performance. This difference has decreased over time, suggesting that it is affected by cultural factors.
- There is one particular spatial skill where gender differences have proved stable across different countries and time, and it is probable that research will find others.

Good teachers will already be aware of different approaches to mathematics and the skills they involve, and will address these when needed.

## 5. Stereotype threat and role models

Mathematics is represented in popular culture as a form of rational masculinity that challenges physical and emotional forms of reasoning. Cultural studies research into images and identities in mathematics shows that representations of mathematicians are associated with maleness, Whiteness, middle-classness and heterosexuality. They are allied with heroism and unusual natural intelligence, as in the Bletchley Park codebreakers, but also with fragility and social incompetence. There is a relatively new media image of young, attractive women 'geeks' that contrasts with the old, male image of mathematics (Mendick, Moreau, \& Hollingworth, 2008; Pomerantz \& Raby, 2011) and is played on in youth culture. Mendick et al investigated how such gendered representations of mathematics were used and understood by students through 27 focus groups and 49
interviews with year 11s and undergraduates in England. They found that both male and female students use the strong default representations of male mathematicians but are aware that these are stereotypes. They are ready to accept that popular, attractive women can be mathematicians but mark these women out as unusual. The girls in the study were less likely than boys to identify themselves as good at mathematics, or to identify themselves with the media images of smart young women. This nuanced effect that students are conscious of gendered STEM stereotyping while at the same time reproducing it has also been found in mathematics research in Sweden and the Netherlands (Brandell \& Staberg, 2008), where it becomes significant between ages 15 and 17, and in younger British children's attitudes to science (Archer, Osborne, \& DeWitt, 2012). The researchers' message is that although some girls are willing to prove themselves by publicly aligning themselves with an image of mathematical femininity marked out as different, many more feel distanced by it. They recommend that schools make a much wider range of images available to students: of young and old mathematics users, attractive and not attractive, sporty and not sporty, with a particular focus on users of average ability and career success. Similarly, the most recent findings from work in science suggest that interventions based on the message 'STEM is for girls too' reinforce the existing STEM and gender stereotypes. Instead it should be replaced by a diverse set of images of STEM that focus on STEM being for everyone (CaSE, 2014).

The research above shows how cultural stereotypes, even when challenged, make a difference to how individuals see themselves in relation to mathematics. They have also been found to have an effect on test performance. One study (Nosek et al., 2009) has shown this effect on a large scale using its very large international data bank of individuals' implicit stereotypes - measured by speed and success on sorting activities related to gender STEM stereotypes - and explicit ones, given by strength of agreement with a statement of the stereotype. Averaging these results by country showed that the strength of a country's gender stereotyping correlates with sex differences in TIMSS mathematics and science scores. Implicit stereotypes account for nearly a quarter of the variance, much more so than explicit ones. This study shows that national test performance is affected by social phenomena, but it leaves open how this might operate, for example through affecting preparation for tests or actual test performance.

The mechanism by which stereotypes affect how individuals perform in tests is known as stereotype threat. Psychology experiments show that stereotype threat happens in particular conditions: where there is a population taking a test and there is an underlying stereotype that one population subgroup performs worse on this kind of test. The experiments consist of activating the stereotype for a random half of the population before they take the test. Many small scale experiments have shown that activation reduces the test performance of the stereotypically 'worse' group. The kind of activation that has been found to have an effect varies, but includes watching a TV commercial, ticking a gender box on the test sheet and even writing your name at the beginning rather than at the end of a test (Alcock, Attridge, Kenny, \& Inglis, 2014; D Halpern et al., 2007; Lauer et al., 2013; Maloney, Schaeffer, \& Beilock, 2013). Stereotype threat has been found for women taking
mathematics tests involving simple arithmetic and solving an equation. It has also been found in situations relating ethnicity and sport (Maloney et al., 2013). The psychological explanation is that the affected subgroup have a reaction to the stereotype that takes up working memory in the same way as mathematics anxiety does, and thus affects performance (Maloney \& Beilock, 2012).

Stereotype threat studies have been conducted mainly in labs or in undergraduate education, not with school age children. Meta-analysis has established a significant effect of stereotype threat, but its effect size is not large. For example it does not account for the remaining differences in male and female performance in the US college entry tests (Halpern et al., 2007). It has most effect when stereotypes are not so blatant as to set up a resistance, and for women who have a moderate interest in mathematics. Importantly, it can be reduced by teacher interventions (http://reducingstereotypethreat.org/reduce.html). Some of the messages about how to challenge stereotype threat are complicated: informing girls of how it may affect them before the examination has been found to negate its effect, but then reminding them of gender identity has been found to increase it.

Three recommendations that seem workable are:

- providing external attributions for difficulty in test situations, i.e. emphasising reasons other than gender and natural ability that make mathematics difficult
- giving feedback that communicates high standards and reassures students they will meet them. This appears to signal that students will not be judged stereotypically and that their abilities and "belonging" are assumed rather than questioned
- emphasising an incremental view of ability where success follows effort and failure is expected and remediable. Treating tests as learning challenges that are exciting rather than threatening produces an atmosphere that reduces anxiety and raises girls' performance.

Overall, it is clear that stereotypes of mathematics as masculine have a significant effect on girls' participation. The ASPIRES project traces how early this starts in its survey of 9000 students in England. Girls who reported themselves as 'girly' at 10/11 were less likely to have science career aspirations, and unlikely to persist with them by age $12 / 13$. The Institute of Physics report (Hollins, Murphy, Ponchaud, \& Whitelegg, 2006) reported that participation in physics is highest in schools where stereotypes are treated as out of date rather than a personal challenge to be overcome. This agrees with the recommendations for reducing stereotype threat given above.

It also appears that girls aspiring to take mathematics are more likely to have competitive personality traits than boys choosing mathematics (with a small effect size $d=0.22$ ), and girls as a group are more competitive than boys ( $\mathrm{d}=0.42$ ) (Mujtaba \& Reiss, in preparation, 2013) . Girls as well as boys talk about doing mathematics as proving something about themselves, and for girls this includes a personal challenge to gender stereotypes (Currie, Kelly, \& Pomerantz, 2006; Mendick, 2006; Pomerantz \& Raby, 2011).

The recommendations above suggest downplaying the status that successful girls currently get from doing mathematics, and this raises the question whether this would affect these girls' participation. However the same research on successful girls emphasises the tensions they experience in maintaining that identity if or when external validation falters, for example by grades dropping. They conclude that cultural stereotypes inevitably act on girls doing mathematics to provide a sense of challenge and 'specialness' so that the role of teachers is to mitigate this with evidence of capability and support. These stereotype effects connect with girls' self-concept, discussed further below.

One response to stereotyped images has been to consider the effect of contact with mentors or role models. I have found no research on whether the gender of teachers affects girls' engagement in mathematics. Surprisingly there is little research even at primary level where it has been the focus of media attention. Skelton et al. (2009) found the gender of primary teachers had no effect on their pupils' perceptions of them, and report that studies outside Britain find that matching pupils and teachers by gender has no effect on achievement.

Ofsted recommends that schools invite women scientists to visit with the aim of challenging stereotypes and providing role models (Ofsted, 2011). Evidence from STEM interventions such as a well-established Saturday club with women presenters and facilitators (Watermeyer, 2012) and a 6-week special curriculum unit for year §9 (Archer, DeWitt, \& Dillon, 2014) found that exposure to examples of women in science can have small but lasting effects in promoting resilience amongst girls who already intended to study a STEM subject, and can broaden
 their views of where science can lead. However, these interventions were seen by students as different from school and made little difference to students who already had low STEM aspirations.

The messages from this research are:

- that interventions should use a diverse set of STEM images, that focus on mathematics being for everyone (CaSE, 2014). Instead of juxtaposing existing stereotypes, a wide range of images and examples should be available to students:
of young and old mathematics users, attractive and not attractive, sporty and not sporty, with a particular focus on users of average ability and career success.
- that girls may experience anxiety in mathematics tests that is attributable to stereotype threat, and that teacher encouragement and feedback helps to mitigate this. (Without any research evidence, it seems likely that this would be just as helpful for low-achieving boys who perform against the stereotype of 'gifted mathematicians').
- that exposure to examples of women in science broadens girls' views about where mathematics can lead, but does not, by itself, change low STEM aspirations.


## 6. Mathematics self concept

Students' mathematics self-concept is understood as their perceptions of themselves as good or bad at mathematics, and thus differs slightly from their actual prior attainment or from confidence which is less tightly defined and has more of an emotional aspect. It is well established that boys on average report a higher mathematics self-concept than girls and that students with a high self-concept are more likely to continue with mathematics. This effect is independent of actual attainment. This has been shown among year 11s (Brown et al., 2008), mathematics AS-level students (Pampaka, Kleanthous, Hutcheson, \& Wake, 2011) and it persists into later STEM careers (Heilbronner, 2013). The UPMAP project adds to these findings for self-concept, giving a medium effect size of gender on self-concept ( $\mathrm{d}=0.4$ ). In considering girls only, the effect size of self-concept on STEM aspiration is large ( $\mathrm{d}=0.8$ ) with little difference between year 8 s and year 10s. (Unfortunately the paper does not provide the comparison effect size for boys' aspiration groups). UPMAP also examined retrospective self-concept on the level of mathematical tasks, asking year 8 and 10 girls and boys to rate their certainty that each answer they have given is correct (Mujtaba \& Reiss, in preparation). Analysing the four subgroups of boys/girls with high/low aspirations shows that both groups of boys rated their performance significantly higher than the high aspiration girls, although in fact this group of girls performed as well as the high aspiration boys and significantly better than the low aspiring ones. This suggests that even when girls have a high self-concept and aspiration to take mathematics they are likely to undervalue their day-to-day performances. A further analysis that examined the accuracy of the match between task performance and retrospective self-concept found that boys generally overestimated and girls underestimated their performance compared to average perceptions of each task, with a small effect size (Sheldrake, Mujtaba, \& Reiss, 2014). By year 10 boys' calibration accuracy correlated with their self-concept in mathematics while girls' did not: they still undervalued themselves. This matters because both accuracy and self-concept are correlated with intention to continue and girls are not experiencing the motivating effect of accurately judging their own performances.

The message from this research is that girls' low self-concept in mathematics is an important factor in low participation, that needs to be tackled at a task level for individuals in lessons, and by providing accurate messages about girls' and boys' similar abilities in mathematics. As discussed above, the ASPIRES project has shown that exposure to role models of very able women mathematicians and scientists does not raise most girls' selfconcept and may accentuate differences (Archer et al., 2012): they need also to experience more moderately successful women with more average prior attainment.

I have found two recommendations for teachers within the research. The first draws on Carol Dweck's theory of mastery. A US research project spent ten days in each of 65 grade 6 classrooms (i.e. a large scale project) looking for teaching strategies that fostered selfconcept in mathematics by discouraging self-handicapping strategies such as off-task behaviour, avoiding seeking help and avoiding novel approaches to mathematics task (Turner et al., 2002). They found that successful teachers paid attention to preparing students for mathematical challenges. They used a complex mix of strategies to prepare students cognitively and no one style could be identified. For example, they used tighter or looser teacher-control strategically and responsively in their lessons to emphasise important points and promote understanding. However a common theme was their motivational strategy: using humour and acknowledgement of emotions around difficulty. In contrast, teachers who conveyed a high demand for correctness in mathematics but provided little cognitive or motivational support during lessons increased students' self-handicapping behaviours such as dependency. Although originally focused on younger children, this is relevant to A-level teaching because of the need for A-level students to work independently and their difficulties in doing so. The research concludes that self-concept in mathematics is fostered when teachers move responsibility for understanding to the learner, and treat this as a back-and-forth negotiation of mathematical meaning and of motivation. There is no specific reference to gender in these findings, but it directly addresses behaviours related to spiralling self-concept and thus adds to the advice about promoting girls' engagement. Hollins et al. (2006) report for the Institute of Physics also considers teachers' assessment of STEM answers as right and wrong. Their case study interviews suggested that girls preferred to offer responses to teachers' questions that left the way open for other students to contribute. They recommend that teachers should treat students' expressions of uncertain responses in mathematics and science not as showing lack of understanding but, as an invitation to discuss how students individually, or as a group, could establish a more certain response.

The second recommendation is again from UPMAP: the most important influence on girls' participation in mathematics is the advice and encouragement they receive. High aspiring girls received significantly more advice/pressure to follow STEM courses than low aspiring girls (with a large effect, $\mathrm{d}=1.3$ ), more than boosting a grade at GCSE, and more influential than girls' self-concept (Mujtaba \& Reiss, in preparation). The power of encouragement may be just as important for boys (the difference between aspiration groups is not reported), but girls receive somewhat less of it: less advice-pressure to study mathematics ( $\mathrm{d}=0.14$ ) and less home-support for achievement in mathematics $(d=0.24)$ than boys. In support of this
finding, interviews with male and female undergraduates with an A-level in mathematics show the importance of having someone, whether a teacher or family member, who offered personal encouragement to choose a STEM subject (TISME, 2013).

The messages from this research are:

- girls from year 8 onwards consistently under-rate their performance in mathematics, both overall and on particular tasks. Since students with a high self-concept are more likely to continue with mathematics, this reduces girls' participation at 16.
- role models in mathematics need to include moderately successful men and women with more average prior attainment.
- there is evidence for younger pupils that teaching that pays attention to preparing students cognitively and motivationally for challenge promotes students' selfconcept; while teaching that emphasises right and wrong answers without motivational support promotes self-handicapping strategies.
- advice and encouragement is effective in mediating the effect of girls' lower selfconcept on participation.


## 7. Different ways of being mathematical

Quantitative studies such as UPMAP and Brown, Brown and Bibby's 2008 paper have shown that girls have different perceptions of mathematics lessons and this is also a significant factor affecting participation. There is some evidence that teachers treat boys and girls differently in mathematics lessons. For example, the 'smart' girls in Pomerantz and Raby's US study (2011) report that teachers praise boys for lesser achievements. Warrington and Younger's four-year study of boys' underachievement (2000) in England showed that teachers valued boys' work for speed, sparkle and laziness, which they found exciting and an indicator of hidden potential, rather than the consistency they attributed to girls. If these teacher beliefs are conveyed into classroom messages, there must be a negative effect on girls.

There is some agreement in the research that many girls want a different kind of mathematics classroom, one that emphasises discussion and a quest for understanding (Boaler et al., 2011; Brandell \& Staberg, 2008; Hollins et al., 2006). Top set year 11 classrooms are most relevant for Further Mathematics A-level continuers and these are found to have a consistent emphasis on speed, accuracy, competition and lack of discussion that marginalises many girls, although not the most competitive ones (Bartholomew, 2005; Solomon, 2009). Among undergraduate mathematicians, Solomon found that young women who could achieve in tests without really understanding felt that they were not really good at mathematics, while young men in the same position were more satisfied with the evidence from the tests. Solomon also found that top set classrooms gave opportunities for exploratory mathematics that engaged both boys and girls (although this was related to GCSE investigation coursework). It is probably not helpful to focus on a
quest for understanding as a gender difference since the above studies agree that low selfconcept boys are also found to want more opportunities to develop understanding. Year 11 classrooms that use formative assessment, exploration and discussion, that do not proceed too quickly to assessment and that allow students to master concepts in depth are supportive for girls and boys.

The research on mathematical performance in section 4 highlighted that mathematics involves a range of skills and making connections between them. Leone Burton's research with professional mathematicians showed that they combined different ways of reasoning: visual (thinking in pictures, often dynamic), analytic (thinking symbolically, often formalistically) and conceptual (thinking in ideas, classifying) (Burton, 2004). Year 11 and 12 curriculum topics such as trigonometry, graph sketching and calculus demand that students connect these types of reasoning if they want to develop strategies for non-routine problems. Teaching that recognises and even emphasises multiple strategies for solving problems and how to move between multiple representations of mathematical ideas can affect students' participation by deepening understanding. This affects all students, but may have particular benefits for girls' participation because negotiation and comparison of different perspectives is a feature of girls' interactions in friendship groups (Hey, 1997) that is reported as increasing lesson enjoyment for girls (Hollins et al., 2006), which is known to be less than for boys and significant for participation.

There is also some evidence from longitudinal studies of very high performers in the US that students who have higher scores on visual reasoning are more likely to continue with mathematics than others with the same mathematical performance (Wai, Lubinski, \& Benbow, 2009). There is no overall gender difference on the visual reasoning tests, and researchers suggest that developed visual reasoning skills may not influence choice directly, although it may increase students' self-concept and enjoyment of mathematics. Halpern et al. point out the distinction between the 3-D mental rotation tasks in which there is a known gender difference and the more familiar visual skills of 2-D representation, where there is none ( $D$ Halpern et al., 2007). However, they recommend that teachers should be aware that on average girls will need more support in 3-D visualisation. Mathematics and further mathematics A-level include such problems, for example visualising distance between skew lines or involving 3-D diagrams in co-ordinate geometry or mechanics. Teaching that emphasises only symbolic and numeric approaches is restrictive and does nothing to prepare more girls to participate.

The messages from this research are:

- girls and boys have different perceptions of mathematics lessons; many girls (and some boys) dislike an emphasis on speed and closed answers.
- When girls feel they do not understand mathematics, this combines with sociallybased feelings of exclusion to affect their participation. Teaching that allows group discussion and acknowledges multiple strategies for understanding and solving problems fosters understanding. Classrooms that use formative assessment,
exploration and discussion, that do not proceed too quickly to assessment and that allow students to master concepts in depth are supportive for girls and boys.
- Teachers should be aware that students have different capacities in 3-D visualisation, and that students can improve their skills by practice. On average this would help girls more than boys.


## 8. Organising learning

There are some messages from the research that suggest a school-level approach to encouraging girls' participation. One question here is whether single sex schools have a higher proportion of girls taking mathematics A-level. This is undoubtedly true, with $32.3 \%$ of the A-level cohort in single-sex girls' schools taking mathematics in 2013 compared to the $20.4 \%$ participation of girls in all schools (and $5.0 \%$ compared to $2.4 \%$ for Further Mathematics) (DfE, 2014a). However there are effects of selection that come into play. DfE data from 2011 took GCSE grades into account and found that this higher proportion came from more girls with a B grade continuing to take A-level (Department for Education, 2011). The UPMAP project controlled for key variables including socioeconomic status and attainment and found that high aspiring girls in single sex schools in years 8 and 10 had more positive perception of mathematics lessons than high aspiring girls in mixed schools (small effect size 0.12 ) and had more competitive personalities than girls from mixed schools (small effect size 0.17) and there was no significant difference in intention to study attributable to the type of school. This agrees with Australian research that found female engineering undergraduates from single-sex schools ascribed their choices to a high general self-concept rather than a particular STEM message (Tully \& Jacobs, 2010). It suggests that, if there is a particular lesson to be learnt from single sex schools, it concerns what single sex schools are doing to engage a broader range of year 11 girls in mathematics and to build up their self-concept.

Once schools have built a momentum of encouraging girls into A-level, research suggests it may be sustainable. Girls participate significantly more in schools that have larger A-level mathematics cohorts, although these may not necessarily be large schools (Department for Education, 2011; Tripney et al., 2010). Girls are more likely to aspire to study mathematics when their school peers around them have high aspirations (Mujtaba \& Reiss, in preparation) and enjoy mathematics (Brown et al., 2008) and when their friends choose the same subjects (Rudasill \& Callahan, 2010) and this phenomenon is more pronounced than for boys.

Schools should also consider the combinations of A-levels taken by boys and girls. The most common pairings of A-levels with mathematics are Chemistry, Physics and Biology (Hillman, 2014) and girls are much less likely than boys to take Physics, almost as likely to take Chemistry and more likely to take Biology (DfE, 2014a; Royal Society, 2008). Girls are also more likely to take non-science subjects with mathematics, following their more
balanced GCSE profile (Bell, Malacova, \& Shannon, 2003). Girls taking A-levels in Business Studies, Economics, Geography, Psychology and Sociology would all benefit from studying mathematics as a supporting subject because of the mathematical content of these courses (Kathotia, 2012). Promoting mathematics in a range of combinations is thus a means of recruiting girls. There is a common sense argument that concentrating on three A-levels is better than taking more. This is not as strong when you consider subjects that reinforce each other. In fact, taking four or more other A-levels is associated with raised odds of gaining $A$ or $B$ in a STEM subject even after prior attainment in the subject is controlled for (Department for Education, 2011). These figures include the mutual reinforcement of Mathematics and Further Mathematics, and there is support outside FMSP for the view that the existing interconnectivity of mathematics and further mathematics modules has increased students' confidence and boosted participation (Hillman, 2014; Smith, 2010).

The messages from this research are:

- to promote mathematics to groups of girls rather than to individuals
- to provide examples and advice in mathematics that relate to its relationship with subjects such as Biology , Chemistry Business Studies, Economics, Geography, Psychology and Sociology.


## 9. Giving girls reasons and support to study mathematics

In this final section reviewing ways to increase participation, I discuss recommendations on interventions that aim to motivate girls to study mathematics.

Perceived utility is one of the key reasons for students choosing mathematics, and this is normally taken to mean a sense that mathematics is useful for preparing for certain careers, accessing desired courses and solving everyday problems. Both girls and boys report the utility of mathematics as a reason for choice, but boys more so. The large scale research studies share this finding (Halpern et al., 2007; Hodgen, 2013; Mujtaba \& Reiss, in preparation; Tripney et al., 2010). Conversely, students who do not consider mathematics useful are less likely to study it, and this is accentuated for girls. Brown et al's (2008) research found that $30 \%$ of students with grades A, B or C at GCSE had chosen not to study mathematics because it would not be useful to them in later life, the second most common reason after difficulty.

The same studies show that girls who choose mathematics are more inclined to report that they do so out of interest or an appreciation of its intrinsic value, and that more girls than boys find mathematics uninteresting. A study of relatively lower attaining girls taking AS level mathematics found that girls talked about choosing mathematics as a way to escape limitations, and described this as 'just for interest', without the vocational rationale that boys used (Hernandez-Martinez et al., 2008). This complements a similar finding that 'smart' girls
valued mathematics for the power it gave them to make and defer choices rather than any specific vision of a career (Pomerantz \& Raby, 2011). It seems likely that the distinction between choosing for utility or interest is bound up with the different ways in which girls and boys present themselves as aspiring young adults. Evidence that such differences do exist is presented by Archer et al's (2010) study of young peoples' careers talk in urban schools which found that learning for interest is associated with youth and femininity and that learning for earning or practical purposes is associated with responsible, adult masculinity . That is not to say that boys don't learn for interest or girls for utility, as they clearly do, but that their learning tends to be framed by themselves and by others in a way that relates to their gender identity.

It is reasonable to conclude that showing students the many ways in which mathematics could be valuable to them is a way to raise both interest and perceptions of utility. The Institute of Physics reports concerns that more girls than boys choose biological sciences because they intend to enter health and medical professions (Hollins et al., 2006), and mathematics and physics are not sufficiently associated with these careers. This perpetuates a cycle where girls with science aspirations make early choices that result in narrow science options, and then low participation in engineering and computing careers (OECD, 2012) entrenches stereotypes and creates further lack of interest. The report recommends that teachers should not leave it to students or careers advice to make the connections between mathematics and careers but bring a range of examples into everyday teaching as well as promotional initiatives.

The ASPIRES study points out the many positive evaluations of one-off initiatives but also raises some concerns. They note that students' responses to science and mathematics are varied, with some reporting that science is interesting for its feelings of problem solving and clear satisfaction while others cite its relevance to complicated social issues. Short initiatives thus risk attracting some students and alienating others (TISME, 2013). There is evidence that they bolster the aspirations of girls who are already inclined to study science, which may otherwise be at risk (Archer, DeWitt, \& Dillon, 2014).

ASPIRES and UPMAP recommend that departments aim to attract girls to science and mathematics by the time they are 12 or 13 because attitudes form early and take time to change. They warn against messages that insert feminised women into existing mathematical stereotypes because such women, though aspirational, are seen as occupying unusual and precarious positions. This reinforces the message that mathematics is for specialised careers and clever people who love it. Instead they recommend, as above, that we need to broaden the range of examples and images that relate mathematics to people and activities, and take in more diverse and less prestigious careers.

ASPIRES also discusses the interactions of gender and class through the Bourdieuan idea of cultural capital applied to science. 'Science capital' refers to a family's science-related qualifications; understanding, knowledge of related institutions, interest and social contacts. Students who come from families with low science capital and expressed no intention to
study science by the age of 10 were unlikely to change their minds by 14 (ASPIRES, 2013). Class and science capital interact in several ways. Middle class families tend to have more science capital, although this is not necessarily the case. In addition, middle class families are more likely to use websites and official information to support decisions about education and careers. Working class families are just as likely to have ambitions for girls but they more likely to use 'hot knowledge' drawn from families and friends (Ball \& Vincent, 1998). This is relevant for girls' participation because the girls who maintain science or STEMrelated aspirations over time tend to possess high or medium levels of science capital. Families with low science capital do not have the network of support that is needed to sustain girls' engagement in mathematics and science careers. Schools, and the FMSP, have an important role to play in building networks of information for families about the diverse ways in which girls are using the mathematics they learnt in school.

The messages from this research are related to previous recommendations:

- that teachers should provide example in mathematics that relate to a range of STEM careers including architecture, veterinary sciences, health, caring and business; and the teachers should explain their relevance.
- that interventions to interest girls in mathematics should start in the early years of secondary school
- interventions should address families as well as students, and provide examples, information and local contacts that help them feel knowledgeable and comfortable with steps to a STEM career.


## 10. Bibliography

In this bibliography, sources are listed alphabetically with in the following categories: peerreviewed empirical research studies, books or reviews drawing on the authors' prior research, reports commissioned by professional associations, charities or learned societies, and miscellaneous papers cited in my commentary that are not directly relevant to the papers reviewed. Each is divided where appropriate into UK studies first and then international studies.

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